Table 1 Comparison of first variations and eigenvalues from the classic method and the improved calculation

Mode	Parameter	Exact	Classic	Improved
1	$\delta\lambda_1$	0.001694	0.001687	0.001696
	λ_1	0.330121	0.330114	0.330122
2	$\delta\lambda_2$	0.031665	0.031439	0.031598
	λ_2	2.343013	2.342787	2.342946
3	$\delta\lambda_3$	0.010113	0.010111	0.010111
	λ_3	4.901943	4.901941	4.901941
4	$\delta\lambda_4$	0.069529	0.068834	0.069529
	λ_4	5.952137	5.951442	5.952137

but the accuracy is not as good as that of the eigenvalue.[8] The first variation was also obtained by Lü et al.[7]

Numerical Example

A plane truss (Fig. 1) is chosen as the example, with the sectional stiffness for five bars and the lumped nodal masses taken as (5.5, 4.5, 1, 1, 1) and (1, 1). The variations of stiffness and masses are assumed to be (0.01, 0.01, 0.01, 0, 0) and (−0.01, 0). The first variations and the eigenvalues obtained by the classic method and the improved calculation are given in Table 1. The relative errors of the variations are shown in Fig. 2. The result of the improved calculation is in the very good agreement with the exact one. Apart from the third mode, the first variations given by the improved calculation are even better than those with the second variations by the classic method.

A large modification of the truss is considered by multiplying $\delta\boldsymbol{A}$ and $\delta\boldsymbol{B}$ with a factor from 1 through 10. The relative errors of the first variations are shown in Fig. 3. The errors in the first mode increase from 0.0827 to 0.8410%, whereas those of the classic method change from −0.4159 to −4.1834%. The fourth mode shows the biggest difference, as the largest error of the improved calculation is −0.0023% but −9.9982% for the classic method.

Conclusions

An improved calculation of eigenvalue variations has been presented. The eigenvalue approximation from the first variation is a Rayleigh quotient. The variations are different from those in the classic method when there is a variation of $\boldsymbol{B}$, which is the inertia-related matrix in a structural dynamics. Although there are more terms in the variations of eigenvalues, no extra matrix operation is required. The given example confirms that the accuracy of the eigenvalue variations is much better than that obtained with the classic method.

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Numerical Presentation of Eigenvalue Curve Veering and Mode Localization in Coupled Pendulums

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Introduction

THE eigenvalue curve veering and mode localization are the phenomena to show a rapid modal change in dynamic systems.[1–9] It was demonstrated that a mistuned length to a pendulum in weakly coupled pendulums can result in the occurrence of the phenomena.[6,8] This small irregularity is a disorder to the pendulums with a nearly periodic feature. Although their occurrence and orientation have been recognized in the published references, a standard to identify the phenomena must be further studied. This Note provides an attempt to use the derivatives of eigenvalue and eigenvector as the numerical presentation of the phenomena in the weakly coupled pendulums.

Curve Veering and Mode Localization

Two weakly coupled pendulums are shown in Fig. 1. The length of the left pendulum is given a disorder factor γ. The eigenvalue equation of the pendulums is such that

$$\left(\begin{bmatrix} 1/(1+\gamma)+R & -R/(1+\gamma) \\ -R-R\gamma & 1+R \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) \begin{Bmatrix} \varphi_1 \\ \varphi_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{1}$$

where $R = kL/mg$, $\lambda = L\omega^2/g$, and ω is the angular frequency. The eigenvector is normalized as

$$(\varphi_1)^2 + (\varphi_2)^2 = 1 \tag{2}$$

Two pairs of the eigenvalues and eigenvectors can be directly solved and are given as

$$\lambda_{1,2} = \frac{2 + 2R + \gamma + 2R\gamma \mp T}{2 + 2\gamma} \tag{3}$$

$$\{\varphi_1 \quad \varphi_2\}_i = \frac{\{1 \quad S_i\}}{U}, \qquad i = 1, 2 \tag{4}$$

where $T = \sqrt{[\gamma^2 + 4R^2(1+\gamma)^2]}$, $S_i = [1 + (R - \lambda_i)(1+\gamma)]/R$, and $U = \sqrt{(1 + S_i^2)}$.

It was found that the values of γ and k determine the occurrence of the curve veering and mode localization as well as the violent degree of the phenomena.[6,8] As a condition, k must be small so that two eigenvalues are close. Furthermore, the phenomena only occur for a small γ. Giving $k = 0.01$, the eigenvalues and eigenvectors in Fig. 2 are showing the curve veering and mode localization at $\gamma \approx 0$. The existence of the phenomena prevents the standard perturbation analysis to give a precise evaluation of the modal changes. A modified

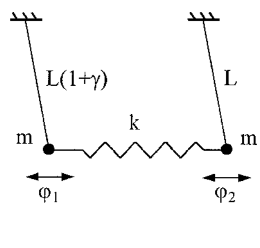

Fig. 1 Weakly coupled pendulums.

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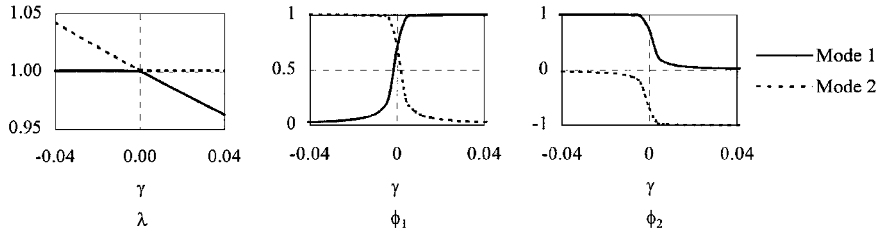

Fig. 2 Eigenvalues and eigenvectors vs γ ($k = 0.01$ and $L/m = 1$).

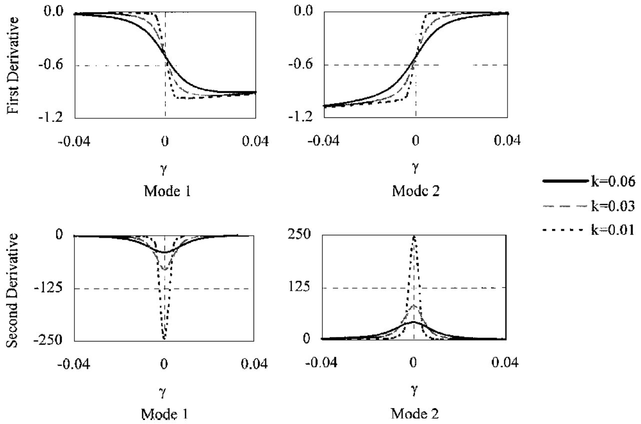

Fig. 3 First and second derivatives of the eigenvalues vs γ ($L/m = 1$).

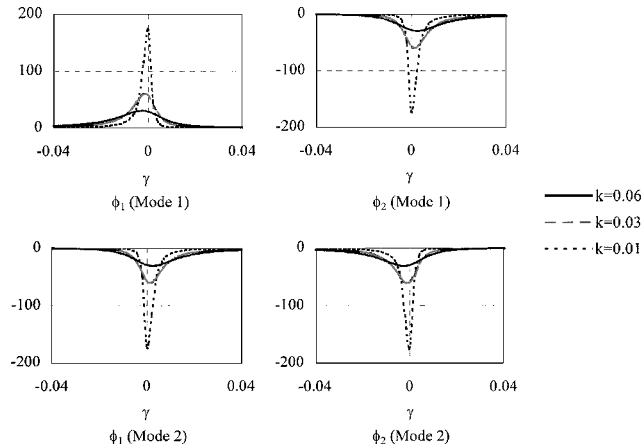

Fig. 4 First derivatives of the eigenvectors vs γ ($L/m = 1$).

perturbation[8] and a singular perturbation[10] were proposed for the pendulums. The generalized method can be used to analyze the perturbation of the disorder.[11]

Numerical Presentation

The rate of change for the eigenvalue or eigenvector is their derivative. From Eq. (3) the first and second partial derivatives of the eigenvalues with respect to γ can be obtained as

$$\frac{\partial \lambda_{1,2}}{\partial \gamma} = \frac{1}{2(1+\gamma)^2}\left(-1 \mp \frac{\gamma}{T}\right) \tag{5}$$

$$\frac{\partial^2 \lambda_{1,2}}{\partial \gamma^2} = \frac{1}{(1+\gamma)^3}\left(1 \pm \frac{1-2\gamma}{2T} \mp \frac{\gamma^2}{2T^3}\right) \tag{6}$$

The first derivatives of the eigenvectors can be written out as

$$\left\{\frac{\partial \varphi_1}{\partial \gamma} \quad \frac{\partial \varphi_2}{\partial \gamma}\right\} = \left\{-\frac{S_i}{U^3} \quad \frac{1}{U} - \frac{S_i^2}{U^3}\right\}\frac{\partial S_i}{\partial \gamma}, \qquad i = 1, 2 \tag{7}$$

where $\partial S_i/\partial\gamma = 1 - [\lambda_i + (1+\gamma)\partial\lambda_i/\partial\gamma]/R$. The derivatives are shown in Figs. 3 and 4 for $k = 0.06$, 0.03, and 0.01, respectively. The trend of the first derivatives of the eigenvalues is similar to the eigenvectors in Fig. 2. With the decrease of k, the peak values of the second eigenvalue derivatives in Fig. 3 increase; however, the derivatives go to a narrow range of γ. A similar feature is found for the first derivatives of eigenvectors. Certainly, the derivatives can go to extremes, when k approaches zero.

By comparing Figs. 3 and 4 to Fig. 2, it is concluded that the second derivative of eigenvalue can be a measure for the curve veering while the first derivative of eigenvector is the presentation of the mode localization. With the help of these derivatives, to identify the occurrence of the phenomena, their distribution and their violent degree should be an easy job. When k is large or γ is far away from zero, the values of derivatives are small, which suggests that the phenomena do not occur. The severe veering and localization are noted from the very large derivatives, when both k and γ approach zero.

The maximum values of the derivatives and their locations, shown in Figs. 5 and 6, can tell more. Slightly different on two modes, the maximum values locate on very small γ either positive or negative, except $\gamma = 0$. The location of γ varies to different k, with a small γ for a small k. For a small and nonzero k, the value of γ becomes very small but is never zero.

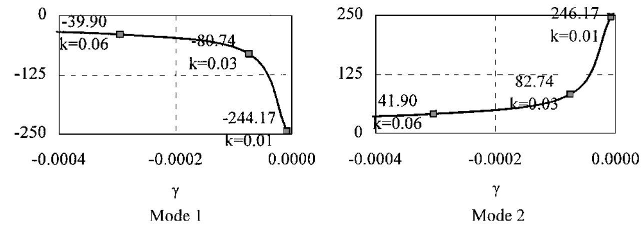

Fig. 5 Maximum values of the second derivatives of the eigenvalues vs γ ($L/m = 1$).

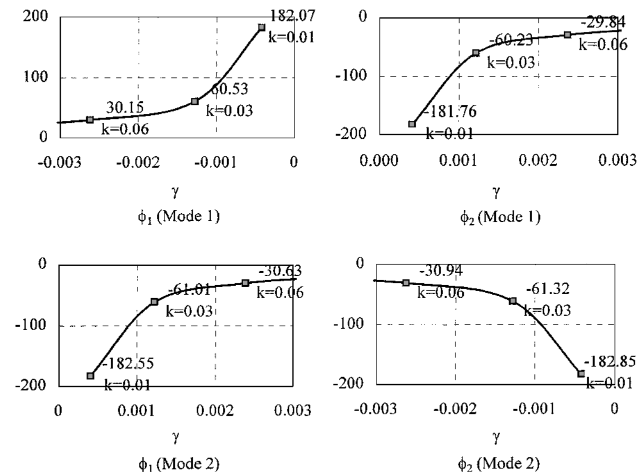

Fig. 6 Maximum values of the first derivatives of the eigenvectors vs γ ($L/m = 1$).

Conclusions

It is demonstrated in the weakly coupled pendulums that the first derivative of the eigenvector is a numerical presentation of the mode localization, whereas the second derivative of the eigenvalue must be used for the curve veering. Violent veering and localization are on the locales where the large values of the derivatives are found. The maximum values of the derivatives come from a very small value of the factor γ, which is never zero as the stiffness k approaches zero. Therefore, the most severe curve veering and mode localization are on these points, which are slightly different from one mode to another.

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Genetic Algorithms for Optimization of Piezoelectric Actuator Locations

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Introduction

OPTIMAL placement of sensors/actuators has drawn significant attention recently due to its importance in many applications such as shape control, vibration control, acoustic control, buckling control, aeroelastic control, and health monitoring of structures. Various methods have been used to address this issue, including the method of placing piezoelectric actuators in the region of high average strain and away from areas of zero strain,[1] heuristic integer programming,[2] tabu search,[3] simulated annealing,[4] genetic algorithms,[5–7] etc.

Genetic algorithms (GAs) have attracted considerable attention due to their ability to solve large complex optimization problems. In this Note, two versions of GAs, GA version 1 and GA version 2, were used to solve two kinds of difficult, computationally intensive, combinatorial, and continuous large-scale optimization problems. The two problems are related to finding both a set of optimal locations and a set of corresponding optimal voltages for 30 piezoelectric actuators, from a maximum possible 193 candidate locations, with more than 1.28×10^{35} possible solutions, which will yield the best correction to the surface thermal distortions of a thin hexagonal spherical primary mirror (Fig. 1a) subjected to four different thermal distortions. In the first problem, a set of actuator locations was obtained for each of the four thermal distortions. In the second problem, only one set of actuator locations effective for all of the four thermal loads was determined. The latter is a more challenging, multicriterion optimization problem. A laminated triangular shell element[8] was used to model the mirror (Fig. 1b). The performance of the GAs and results of comparing with DeLorenzo's algorithm are presented.

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GAs

GAs[9,10] are robust stochastic global optimization techniques based on the mechanism of natural selection and natural genetics. GAs were invented by Holland in the 1960s.[11] They are population-based search algorithms with selection, crossover, mutation, and inversion operations. The advantages and disadvantages of GAs were presented in Ref. 7.

In this study, two versions of GAs, GA version 1 and GA version 2, developed by the present authors from Carroll's FORTRAN GA driver,[12] have the following elements in common.

1) The encoding scheme is the same: the direct mapping, where the string would contain a 1 or 0 in the bits corresponding to the presence or absence of actuators.

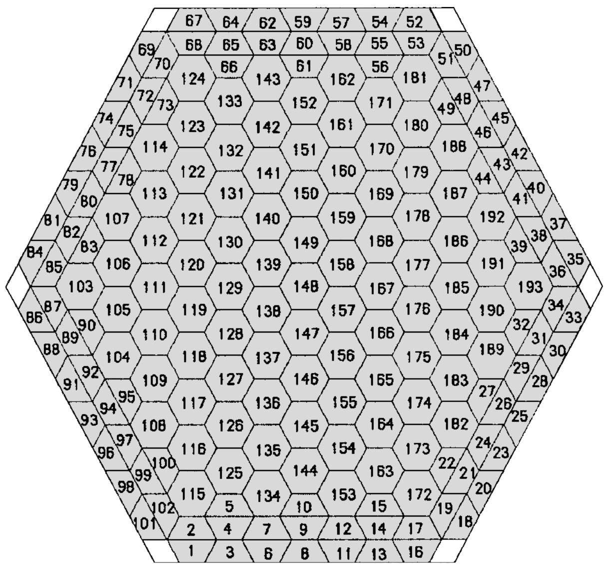

Fig. 1a Piezoelectric actuator candidate locations.

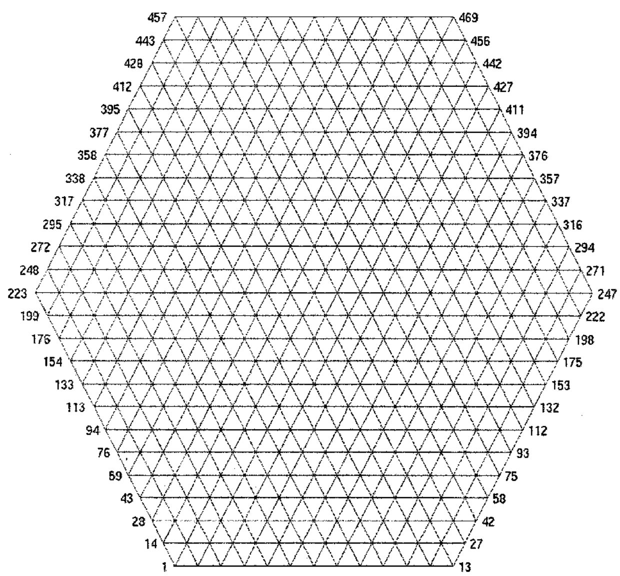

Fig. 1b Finite element mesh.